Answers

1. **E, D, A, B, C**

2. **D**

3. $\frac{3}{a+b+1}$

4. **$10\sqrt{13}$ or 36.056**

5. $\frac{2}{3}$

6. **(5,2), (11,10), (5,-2), (11,-10), (0,0)**

Nassau Replacement Problems – Solutions

1. The areas are A) 25π or about 78.5; B) 30π, or about 94.25; C) 98; D) 48; and E) is a parabola which is clearly contained in a rectangle which is 4 by 6, so the area is less than 24. So the order is **E, D, A, B, C**

2. Only one of these can be true, so the other four must be false. Note that they can't all be false, since than E would be true, a contradiction. Hence **D** is true.

3. $\log_2 30 = \log_2(2\cdot 3\cdot 5) = \log_2 2 + \log_2 3 + \log_2 5 = 1 + a + b$.

   Thus, $\log_{30} 2 = \dfrac{1}{1+a+b}$. Now, $\log_{30} 8 = \log_{30}\left(2^3\right) = 3\cdot\log_{30} 2$

   and so $\log_{30} 8 = \dfrac{\mathbf{3}}{\mathbf{1+a+b}}$

4. Triangle ABC can be thought of as having base AB of length 10, and the altitude is the distance from C to the line containing AB, i.e. $2x + 3y = 8$. There are several methods for getting the distance from a point to a line, which in this case is $2\sqrt{13}$.

   So the area is $\left(\dfrac{1}{2}\right)(10)\left(2\sqrt{13}\right) = \mathbf{10\sqrt{13}}$, which is about **36.056**

5. Method 1: The first choice is a red, so the marble must be from either the red-red box or the green-red box. But it is twice as likely to have been chosen from the red-red box, thus the probability is 2/3

Method 2: Suppose the marbles were all distinguishable. Suppose the red-red box contains marbles R1 and R2, the green-green box contains G1 and G2, and the mixed box contains R3 and G3. The sample space for choosing one from a box and then the other consists of the six items R1-R2, R2-R1, G1-G2, G2-G1, R3-G3, and G3-R3. But the condition that the first one is red limits the sample space to the three items R1-R2, R2-R1, and R3-G3. In 2 of the three cases, the second is red, hence **2/3**

6. Factoring, we get $\left(x^2+y^2\right)\left(x^2-y^2\right) = 21\left(x^2+y^2\right)$. Now there are two cases.

Case 1: $x^2 + y^2 = 0$, which is a correct solution. But this only true if **(x,y) = (0,0)**

Case 2: $x^2 + y^2 \neq 0$, in which case we divide by that factor and get $x^2 - y^2 = 21$.

Now, $(x + y)(x - y) = 21$. Since $x \geq y$, $x - y \geq 0$. But they can't be equal, since $0 \neq 21$, So $x > y$. and $x - y > 0$, so $x + y > 0$ since their product is positive. So there are four possibilities. The values of $x + y$ and $x - y$ must be, respectively, 21 and 1, 7 and 3, 3 and 7, or 1 and 21. Solving each system gives the four ordered pairs **(5,2), (11,10), (5,-2), (11,-10)**, all of which work. So there are five such ordered pairs.