

Solutions
Contest #6

25. Let A_t = number of grams after t days, so $A_t = 180 \cdot \left(\frac{1}{2}\right)^{\frac{t}{15}}$ and we want $A_t = 5$.

$$5 = 180 \cdot \left(\frac{1}{2}\right)^{\frac{t}{15}} \text{ and } \frac{1}{36} = \left(\frac{1}{2}\right)^{\frac{t}{15}}. \text{ Take the log of each side and get } \log\left(\frac{1}{36}\right) = \frac{t}{15} \cdot \log\left(\frac{1}{2}\right)$$

Solving, we get $t \approx 77.5$.

26. Method 1: $y = \frac{1937-19x}{23} = 84 + \frac{5-19x}{23}$ and $5-19x$ must be a multiple of 23, and also y must end up being positive and so must x . Solving, the only solutions are $(85,14)$, $(62,33)$, $(39,52)$, and $(16,71)$.

Method 2: Compute mod 19 and get $4y \equiv 18 \pmod{19} \equiv 56 \pmod{19}$, so $y \equiv 14 \pmod{19}$.

Now y must be 14 or 33 or 52, etc. and check which values give positive x .

Note : The table feature of your calculator is very useful for this kind of problem.

27. If K is odd, all powers are odd. If $K = 0$, all powers end in 0. If $K = 6$, all powers end in 6.
Only 2, 4, 8 work.

28. The midpoint of the diagonal connecting $(-1,5)$ and $(7,35)$ is $(3,20)$, the center of the square.
The other endpoints will be at a perpendicular direction, so are 15 right or left, and 4 up or down.
So they are at $(-12,24)$ and $(18,16)$.

29. The number of possibilities is $\frac{6^4}{2} = 648$. To get a sum of 6, it can be one 3 and three 1's (4 ways),
or two 2's and two 1's (6 ways), so the probability is $\frac{10}{648} = \frac{5}{324}$.

30. The slope of \overline{CA} is $-\frac{3}{2}$ so the slope of the altitude is $\frac{2}{3}$ and an equation of the altitude
is $y-14 = \frac{2}{3}(x-6)$. An equation of \overline{CA} is $y = -\frac{3}{2}(x-4)$. Solving for y , we get $y = \frac{114}{13}$.