Nassau County Interscholastic Mathematics League

Contest # 4 Answers must be in simplest exact form, unless otherwise noted 2005-2006

Team Problems, 35 minutes, Calculators permitted

T1. If the lengths of two opposite sides of a square are increased by 6 units each, and the lengths of the other two opposite sides are decreased by 3 units each, the new rectangle that is formed has an area that is 12.5% greater than the area of the original square. Find the length of a side of the original square.

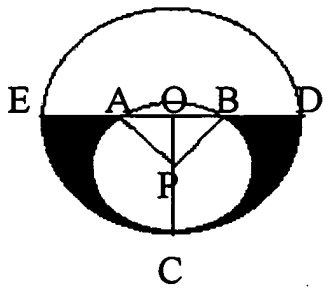
T2. In regular dodecagon ABCDEFGHIJKL, a regular hexagon is formed by connecting, in order, vertices ACEGIK. If the length of a side of the regular hexagon is $9\sqrt{2}$, find the area of the dodecagon.

T3. If $3 \cdot 9^x + 12^x = 2 \cdot 16^x$ and $x = \log_{(\frac{3}{4})} y$, find $y$.

T4. On a test consisting of five multiple-choice questions, each question having four possible answers, and five true-false questions, a student guesses at all the answers without reading the questions. What is the probability that the student will answer exactly seven questions correctly? (Express your answer as a fraction in lowest terms)

T5. Two exterior elevators on an observation tower each travel a distance of 600 feet in opposite directions, each at a different uniform speed. It takes the ascending elevator 14 seconds longer to travel the 600 feet than the descending elevator. If each of the elevators is 13.8 feet in height, and the time required for the elevators to completely pass each other is 0.48 seconds, find the speed of the faster elevator, in feet/second.

T6. The area of circle P is $72\pi$. Circle P is tangent to circle O at C, and diameter $\overline{DE} \perp \overline{OC}$. If AB = $\sqrt{216}$, find the area of the shaded region in terms of $\pi$.




Answers: T1. 12 T2. 486

T3. $\frac{2}{3}$ T4. $\frac{55}{2048}$

T5. 37.5 T6. $33\pi - 18\sqrt{3}$