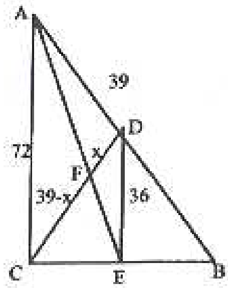
Nassau County Interscholastic Mathematics League
Solutions, Contest 4

#19. The square root of 600 is close to 25. Thus, the two pages across from each other in the middle of the pamphlet are numbered "24" and "25". From page 1 through page 24, there are 24 pages. The number of pages in the pamphlet is twice that, 48.

#20. In right △ ACB below, C is the orthocenter. $\overline{CD}$ is the median to the hypotenuse and its length is 39, one-half the length of the hypotenuse, 78. Medians $\overline{AE}$ and $\overline{CD}$ intersect at F, which is the centroid of △ ACB. When the midsegment $\overline{DE}$ is drawn, DE is 36 since the segment joining the midpoints of two sides of a triangle is half as long as the third side, and △ ACF and △ EDF, are similar. If we let FD = x and CF = 39 – x, then $\frac{36}{72} = \frac{x}{39-x}$. So, x = 13 and CF = 26.



#21. The area of the circle is $\frac{16}{\pi^2} = \pi r^2$; So, $r = \sqrt{\frac{16}{\pi^3}} = \frac{4}{\pi\sqrt{\pi}}$; Thus, $d = \frac{8}{\pi\sqrt{\pi}} \approx 1.4367$.

#22. Factoring yields $\frac{(x^3+1)(x^3-1)}{x^3(x^2-1)+1(x^2-1)} = \frac{21}{5}$; $\frac{x^2+x+1}{x+1} = \frac{21}{5}$; $5x^2 - 16x - 16 = 0$.

$(5x+4)(x-4) = 0$; $x = -\frac{4}{5}$ or 4.

#23. The acceptable combinations are 6 valencias, 5 valencias and 1 temple, 5 valencias and 1 navel, 4 valencias and 2 temples, 4 valencias, 1 temple, and 1 navel, and 3 valencias and 3 temples.

So, the probability is

$$\frac{{}_8C_6 + 6\cdot{}_8C_5 + 4\cdot{}_8C_5 + {}_8C_4\cdot{}_6C_2 + {}_8C_4\cdot 6\cdot 4 + {}_8C_3\cdot{}_6C_3}{{}_{18}C_6} = \frac{317}{1326} \approx 0.2391$$

#24. Draw $\overline{AE}$ and reflect the figure over $\overline{BD}$, producing an isosceles trapezoid. Draw the altitude of the trapezoid from A to F, F being a point on $\overline{DE}$ between D and E. In right △ AEF, AE = 37, EF = 12, AF = 35. Let the reflection of E over $\overline{BD}$ be E'. In right △ AFE', AE' = 91, AF = 35, and E'F = 84. Let AB = FD = x. Then, 2x + 12 = 84 and x = 36.